Lecturer: Shrihari Sridharan
MAT404/MSM504 Topology
End-semester Examination

Max Score: 50 | April 28, 2026 | Duration: 180 minutes

General Instructions

- Please read these instructions carefully, before reading the questions in this question paper.
- This question paper contains four parts; three mandatory parts, namely Part A, Part B and Part C; and one optional part, namely Part D.
- Part A (8 ∗ 1)
  - This mandatory part contains **eight** incomplete statements.
  - You are required to complete the statements with the best word possible or the best phrase.
  - Each correct completion carries **one** point.
- Part B (6 ∗ 2)
  - This mandatory part contains **six** statements.
  - You are required to determine the veracity of the statement and declare True or False with justification.
  - A full proper justification may include a proof or a counter-example, as the case may be.
  - Merely stating True or False without any justification shall not fetch you any points.
  - Each correct declaration along with a proper justification carries **two** points.
- Part C (5 ∗ 6)
  - This mandatory part contains **six** questions.
  - You are required to answer **any five questions only**.
  - Each full correct solution carries **six** points.
- Part D (1 ∗ 10)
  - This optional part contains **one** question.
  - Any score you obtain from this part shall be added to your total score, only to the effect that the cumulative score does not breach the maximum score of this exam.
  - A full correct solution to the question in this part carries **ten** points.
- Please state clearly any theorems that you may use as part of the solutions, in your answer script.
- **Cryptic answers** without any clarity will **not be evaluated**. You are encouraged to show all your workings in your answer script.
- Preferred writing style
  - You are encouraged to write your solutions to the questions in Part A in the first page of your answer script.
  - Please begin your answers to the questions in Part B from the second page of your answer script and write only two answers per page.
  - Kindly ensure that the examiner can find your answers to Parts A and B in the first four to five pages of your answer script.
  - Please begin your answers to every question you write from Parts C and D in a fresh page.

Questions

Part A

1. The topology defined on a set X that declares every subset of X as an open set is called the ________.
2. Let X be a topological space and $\mathscr{B}$ be a basis for its topology. Suppose $Y \subsetneq X$. Then a basis for the subspace topology on Y is given by ________.
3. A bijective map $f : X \longrightarrow Y$ between two topological spaces is called ________ if both f and f^{-1} are continuous.
4. A topological space X is said to be ________ if every open cover of X contains a finite subcollection that also covers X.
5. A topological space X is said to be ________ if its topology can be generated by some basis that has countable cardinality.
6. Any nonempty connected subspace of $\mathbb{R}$ is necessarily ________.
7. According to Urysohn's lemma, in a normal topological space, continuous functions can separate ________.
8. The map $f : \mathbb{C}^* = \mathbb{C} \setminus \{0\} \longrightarrow \mathbb{C}^*$ given by $f(z) = z^d$ for some fixed $d \in \mathbb{Z}_+$ is a ________.

Part B

9. $\bigcap_{n \geq 1} \left(-\frac{1}{n}, \frac{1}{n}\right)$ is not an open set in $\mathbb{R}$ with respect to the finite complement topology.
10. Let X be a set and $\mathscr{T}_k$ for $k \geq 1$ be a countable collection of topologies on X. Then, $\mathscr{T} = \bigcap_{k \geq 1} \mathscr{T}_k$ is also a topology on X.
11. The identity map $id : (\mathbb{R}, \mathscr{T}_t) \longrightarrow (\mathbb{R}, \mathscr{T}_s)$ is continuous, where $\mathscr{T}_t$ and $\mathscr{T}_s$ denote the trivial topology and the standard topology respectively.
12. Any subspace Y of a connected topological space X is also connected.
13. Any subspace Y of a locally compact topological space X is locally compact in the subspace topology.
14. Suppose $f_i : X_i \longrightarrow Y_i$ for $i = 1, 2$ are covering maps, respectively. Then the map $F : X_1 \times X_2 \longrightarrow Y_1 \times Y_2$ given by $F(x_1, x_2) = (f_1(x_1), f_2(x_2))$ is necessarily a covering map.

Part C

15. Prove: The map $M : \mathbb{R}^d \longrightarrow \mathbb{R}$ given by $M((x_1, x_2, \cdots, x_d)) = \prod_{i=1}^{d} x_i$ is continuous and thus, any polynomial $p(x) = \alpha_d x^d + \cdots + \alpha_1 x + \alpha_0$ where $\alpha_i \in \mathbb{R}$ is a continuous function of a real varaible x.

16. Let $X = \{0\} \bigcup_{n \geq 3} \left\{n + \frac{1}{n}\right\}$. Let $f : \mathbb{R} \longrightarrow \mathbb{S}^1 \subset \mathbb{C}$ be the map given by $f(x) = e^{2\pi i x}$. Prove: The restriction of f on X, *i.e.*, $f|_X : X \longrightarrow f(X)$ is a bijective continuous map between the respective locally compact topological spaces, but f is not a homeomorphism.

17. Define a T_i space for $0 \leq i \leq 6$. Give one example of a topological space that is T_i for every $0 \leq i \leq 6$.

18. State and prove the Tietze's extension theorem.

19. Consider the topological space $X = \mathbb{R}^d \setminus \{0\}$ and the action of the multiplicative group $\mathbb{R}^* = \mathbb{R} \setminus \{0\}$ on X given by $\lambda \cdot x = \lambda \cdot (x_1, \cdots, x_d) = (\lambda x_1, \cdots, \lambda x_d)$. Define a relation on X by setting $x \sim y$ if and only if there exists a $\lambda \in \mathbb{R}^*$ such that $\lambda \cdot x = y$.

    (a) Prove: $\sim$ is an equivalence relation.

    (b) Prove: $X/\sim$ is compact and path connected in the quotient topology.

20. Let $I = [0, 1] \subset \mathbb{R}$. Let $f : X \longrightarrow Y$ be a covering map where X and Y are any two topological spaces. Let $G : I \times I \longrightarrow Y$ be a continuous map. Suppose $G(0, 0) = y_0$ and $x_0 \in f^{-1}(y_0)$. Prove: There exists a unique map $g : I \times I \longrightarrow X$ such that $g(0, 0) = x_0$ and $f \circ g = G$.

Part D

21. Let X be a locally compact Hausdorff space, Y be a normal space and $f : X \longrightarrow Y$ be a covering map such that $f^{-1}(y)$ is a finite set for every $y \in Y$. Prove: Every ordered pair (K, g_K) determines a continuous function $G : X \longrightarrow [0, 1]$ such that $G|_K = g_K$, provided f maps closed sets in X to closed sets in Y. Here, K is a closed subset of X and $g_K : K \longrightarrow [0, 1]$ is a continuous function. What is significance of the hypothesis regarding the finiteness of $f^{-1}(y)$ for every $y \in Y$?