Lecturer: Shrihari Sridharan
Topology
MAT404/MSM504 Quiz I

Max Score: 10	February 02, 2026	Duration: 45 minutes

General Instructions

- This question paper contains three questions.
- The individual marks for every question is as written before the start of the question.

Questions

1. (3 marks) Let $X = \left\{ f : [0,1] \longrightarrow \mathbb{R} \mid f \text{ is a continuous function} \right\}$. Define a distance function between elements in X as $d(f,g) = \sup\limits_{x \in [0,1]} \left| f(x) - g(x) \right|$. For any given $\epsilon > 0$ and $f \in X$, let $B_\epsilon(f) = \left\{ g \in X \ : \ d(f,g) < \epsilon \right\}$. Let τ be a collection of subsets U of X that satisfies the following condition: For every $f \in U$, there exists an $\epsilon > 0$ such that $B_\epsilon(f) \subsetneq U$.
   Is (X, τ) a topological space? Justify.

2. (3 marks) Consider the function $P : \mathbb{R}^2 \longrightarrow \mathbb{R}$ defined by $P(x,y) = xy$.
   Is P continuous? Justify.

3. (4 marks) Let (X, τ_X) and (Y, τ_Y) be any two topological spaces. Suppose that $X = A \cup B$ where A and B are closed subsets. Let $f : A \longrightarrow Y$ and $g : B \longrightarrow Y$ be homeomorphisms such that $f(x) = g(x)$ for every $x \in A \cap B$. Define the function $h : X \longrightarrow Y$ given by

$$h(x) \quad = \quad \begin{cases} f(x) & \text{if } x \in A, \\ g(x) & \text{if } x \in B. \end{cases}$$

   Is h a homeomorphism? Justify.